ELMACON Preparation Session

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1   Problem Set A

1. A man has a rectangular patio in his garden. He decides to enlarge it by increasing both the length and width by 10%. What is the percentage increase in its total area?

    1. _____21%_____

2. Joe has several identical balls and several identical dice. Together 4 balls and 3 dice weigh 37 grams, while 3 balls and 4 dice weigh 33 grams. What is the combined weight, in grams, of one ball and one dice.

    2. ___10 **grams**___

3. Let $x = N + 2 \times N + 3 \times N + \cdots + 100 \times N$. What is the smallest integer N for which x is a perfect square?

    3. _____202_____

4. To make lawn fertilizer, a manufacturer mixes nitrogen, phosphoric acid, and potash in the ratio of $3 : 8 : 17$. If a batch of the mixture contains 6kg of nitrogen, how much potash does it contain?

    4. ___34**kg**___

5. If the number pattern shown below is continued, find the third number in row 10.

$$
\begin{array}{lccccccccc}
n = 0: & & & & & 1 & & & & \\
n = 1: & & & & 1 & & 1 & & & \\
n = 2: & & & 1 & & 2 & & 1 & & \\
n = 3: & & 1 & & 3 & & 3 & & 1 & \\
n = 4: & 1 & & 4 & & 6 & & 4 & & 1 \\
& & & & & \vdots & & & &
\end{array}
$$

5. _____45_____

6. Jane was born on June 30, 1994.Alex was born on June 3, 1995. Find the number of days between their birth dates (not including their birth dates).

    6. ___**337 days**___

7. In the product $P8 \times 3Q = 2730$, the letters P and Q represent different digits from 1 to 9. Find $P + Q$.

    7. _____12_____

8. The digits of 4795 can be rearranged to form different numbers. What is the sum of the largest and smallest numbers that may be formed?

    8. ___**14333**___

9. The number 1 is both the square of an integer and the cube of an integer. What is the next larger number which is both a square and cube of a positive integer?

    9. _____**64**_____

10. Alf rolls two dice and adds the numbers that come up. What is the probability that the sum is 9?

    10. _____1/9_____

11. A dog and a rabbit are 160 meters apart. The dog chases the rabbit. For every 9 meters that the dog runs, the rabbit runs 7 meters. Find the distance, in meters, that the dog must run to catch the rabbit.

    11. ___720 **meters**___

12. Find the whole number N between 0 and 20 for which the following steps will give the output $A = 9$.

  1. Use the starting number N to build a new number M:
    - If N is 9 or smaller, let M be $N + 10$
    - If N is 10 or greater, let M be $N - 5$
  2. Divide M by 16, and call the remainder R.
  3. Multiply R by 3. Call the result your output, A.

  12. _____9_____

13. Find the area of the shaded triangle in the sketch below. (The distance from each point to its nearest neighbors is one unit).
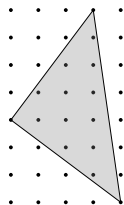
  13. _____**12.5**_____

14. Eve, Jane and Amy have different collections and different clothing.

  1. One of them collects cards
  2. One of them wears a red shirt
  3. Amy collects stamps
  4. The one wearing a white shirt collects rocks
  5. Eve is not wearing a blue shirt and does not collect rocks

  What color is Eve's shirt?

  14. _____**Red**_____

2   Problem Set B

15. Use the following information to find the number N: On a certain island, it never rains all day. If it rains in the morning, it is clear in the afternoon. If it rains in the afternoon, it was clear in the morning. On some days it doesn't rain at all. During a recent period of N days there were 11 clear afternoons, 11 clear mornings, and 10 days when some rain fell.

  15. _____**16**_____

16. In the division problem $(11^{10} - 1) \div 100$, what is the remainder? (Note that 11^{10} means $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$).

  16. _____**0**_____

17. How many members are there in the StayFit club? Clues: There are 40 members who swim, 30 members who play tennis, and 20 members who jog. Five of the members swim and play tennis but do not jog, five swim and jog but do not play tennis, and 5 play tennis and jog but do not swim. Four members do all three activities; all members do at least one.

  17. _____**67**_____

18. There are 9 teams in a soccer league. Each team plays each other team four times. How many games are played?

  18. _____**144**_____

19. Use the following information to find the number N: N is an even three-digit number. N equals the area of a square whose sides are whole numbers; when the length of one side is written down, its digits add up to 4.

  19. _____**484**_____

20. Consider the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many different subsets containing three elements each have elements that sum to 10?
    Background: Both $\{0, 4\}$ and $\{1, 3\}$ are subsets containing two elements that add up to 4: these are the only two-element subsets within this sum. Repeated elements are not allowed in a subset, so $\{2, 2\}$ may not be used. Also, the order of writing makes no difference so $\{1, 3\}$ and $\{3, 1\}$ are considered the same subset.

  20. _____**8**_____

21. A restaurant offers a choice of 3 meat items, 3 vegetable items, and 3 desert items. How many different meals can Jane order, if she chooses 2 different meat items and 2 different non-meat items, of which at least one is a vegetable?

21. _____**80**_____

22. The dimensions in the sketch below are in meters. Find the area of the shaded part, to the nearest whole number of square meters.
Notes: Both figures are circles centered at A. The distance from A to B is 24 meters. The distance from B to C is 2 meters. (The diagram is not to scale).
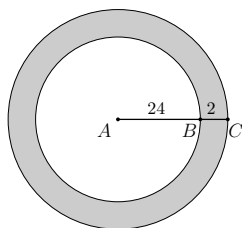
22. _____**314**_____

3   Problem Set C

23. How many numbers less than 100 are divisible by either 3 or 5?
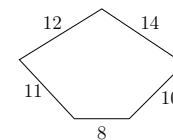
23. _____**46**_____

24. I have 10 quarters in my pocket. I have 20% more dimes than quarters and 25% less nickels than dimes. What is the value of the coins in my pocket.

24. _____**$4.15**_____

25. The sum of $\frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{11}$ is how much less than the sum $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{10}$?
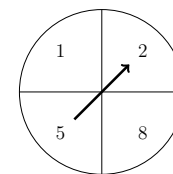
25. _____**10/11**_____

26. Find the average lengths of the sides of the pentagon

26. _____**11**_____

27. A spinner is spun 2 times. What is the probability that the sum of the two numbers is prime?


27. _____**7/16**_____

28. Bob is on the basketball team along with 5 other boys. A basketball team has 5 starting players. How many ways can 5 starting players be selected if Bob must be a starter?

28. ____**5**____

29. I am a whole number. The product of two less than me and two more than me is 165, What number am I?
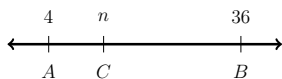
29. ____**13**____

30. The area of the front of the box is 6 times the area of the bottom of the box. What is the volume of the box
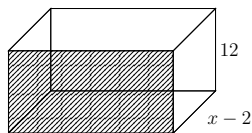
30. ____**432**____

31. What value of n makes AC one fourth of AB?

31. ____**12**____

32. Bob scored $84, 90, 81$ and 93 on his four tests. Bob was caught cheating on his first test and his score was changed to zero. How much did his average drop?

32. ____**21**____

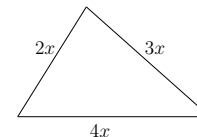33. How many perfect squares are there between 100 and 200

33. ____**4**____

34. What is the least common multiple of 21 and 30?

34. ____**210**____

35. What is the reciprocal of 4.3 to the nearest thousandth?

35. ____**0.233**____

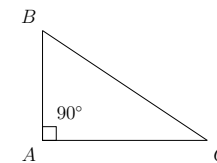36. If the perimeter is 54, what is the value of x?

36. ____**6**____

37. Bob has 80 baseball cards. He sold $\frac{1}{5}$ of them and then gave away $\frac{1}{4}$ of the ones that were left. How many cards does Bob now have?
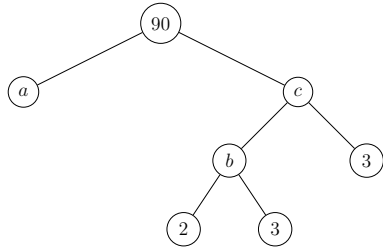
37. ____**48**____

38.

$$\angle A = 90°$$
$$\angle B = \frac{2}{3} of \angle A$$
$$\angle C = ?$$

38. ____**30°**____

39. Find the value of $a + b + c$ for the prime factorization tree.

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        90
       /  \
      a    c
          / \
         b   3
        / \
       2   3
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39. ____**29**____

40. What is 6 days after the day before yesterday if tomorrow is Friday?

40. ____**Monday**____

41. Elvis wants to be fair dividing up the candy he brought to school between Jerry Lee and Little Richard. The problem is that the candy bars are not divided the same way. $1\frac{1}{2}$ pieces of Chocolate Fantasy is the same as $\frac{3}{4}$ piece of Luscious Liplickers. How many Luscious Liplickers would equal 12 pieces of Chocolate Fantasy?

41. ____**6**____

9