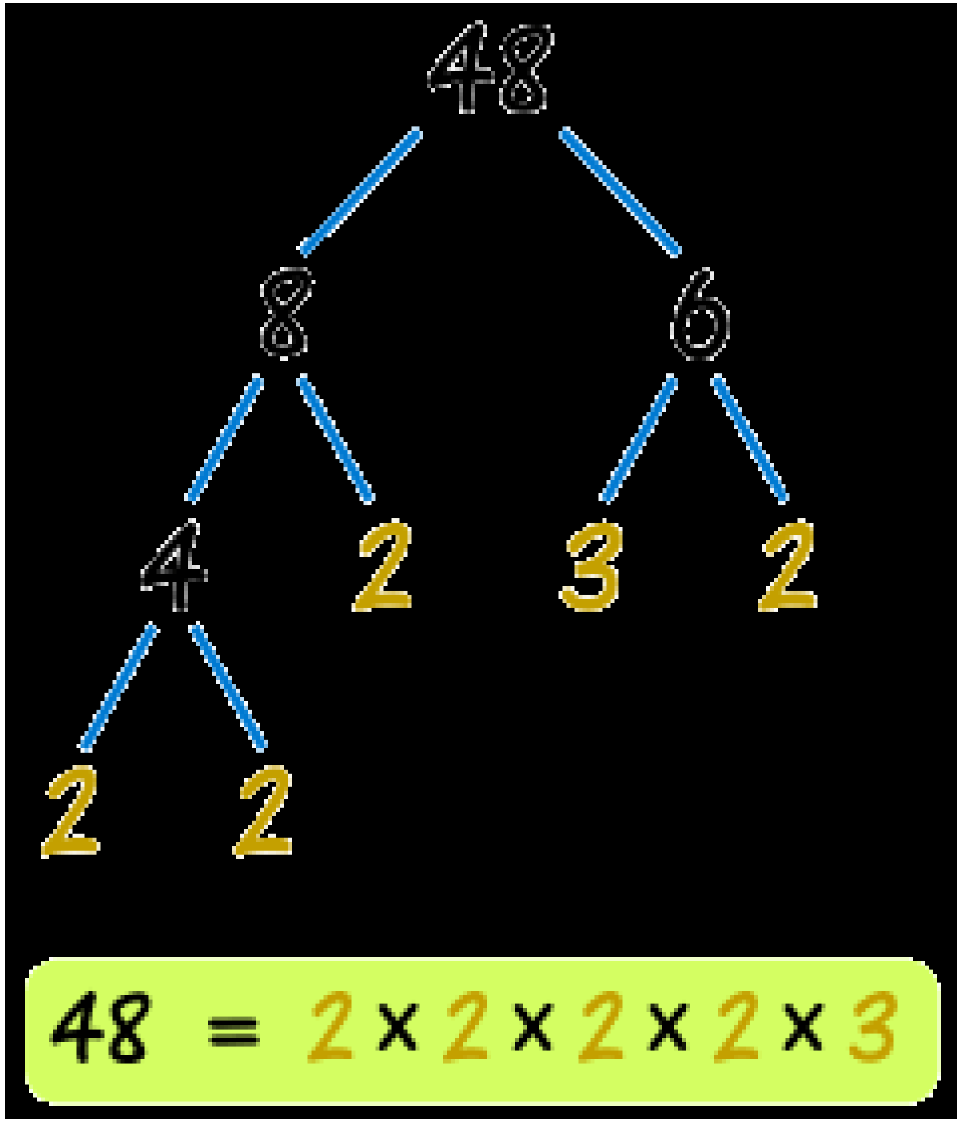
First ELMACON Review
basic concepts

Prime Factorization

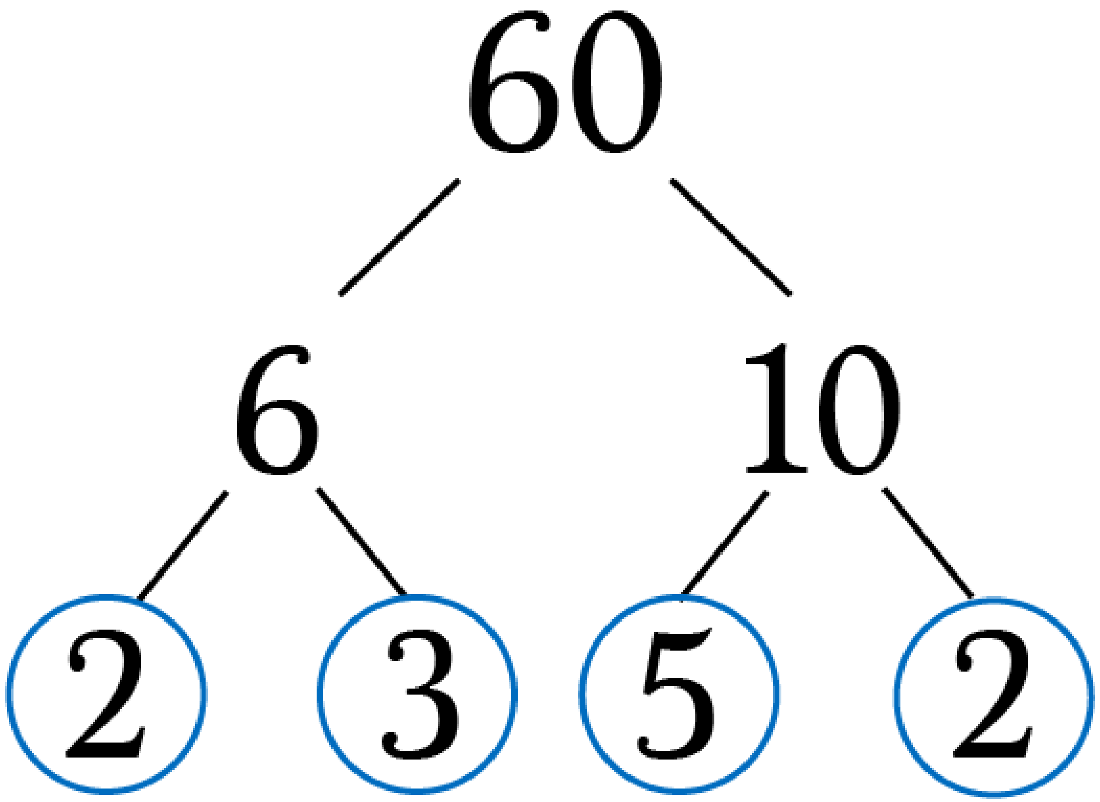
- A Prime Number can be divided evenly **only** by 1 or itself. And it must be a whole number greater than 1.
- •The first few prime numbers are: 2, 3, 5, 7, 11, 13, and 17
- •"Factors" are the numbers you multiply together to get another number:

- "Prime Factorization" is finding which prime numbers multiply together to make the original number.
- **What are the prime factors of 12 ?**
- It is best to start working from the smallest prime number, which is 2, so let's check:
- 12 ÷2 = 6
- Yes, it divided evenly by 2. We have taken the first step!
- But 6 is not a prime number, so we need to go further. Let's try 2 again:
- 6 ÷2 = 3
- •Yes, that worked also. And 3 **is** a prime number, so we have the answer:
- **12 = 2 ×2 ×3**

- "Factor Tree" can help: find any prime of the number, then the factors of those numbers, etc, until we can't factor any more
- **Example: 48**
- **48 = 8 ×6**, so we write down "8" and "6" below 48
- Now we continue and factor 8 into **4 ×2**
- Then 4 into **2 ×2**
- And lastly 6 into **3 ×2**
- We can't factor any more, so we have found the prime factors.
- Which reveals that **48 = 2 ×2 ×2 ×2 ×3**
- (or **48 = 24×3**using exponents)



Here is the prime factorization of 60.
Can this help us to find out how many factors 60 has?

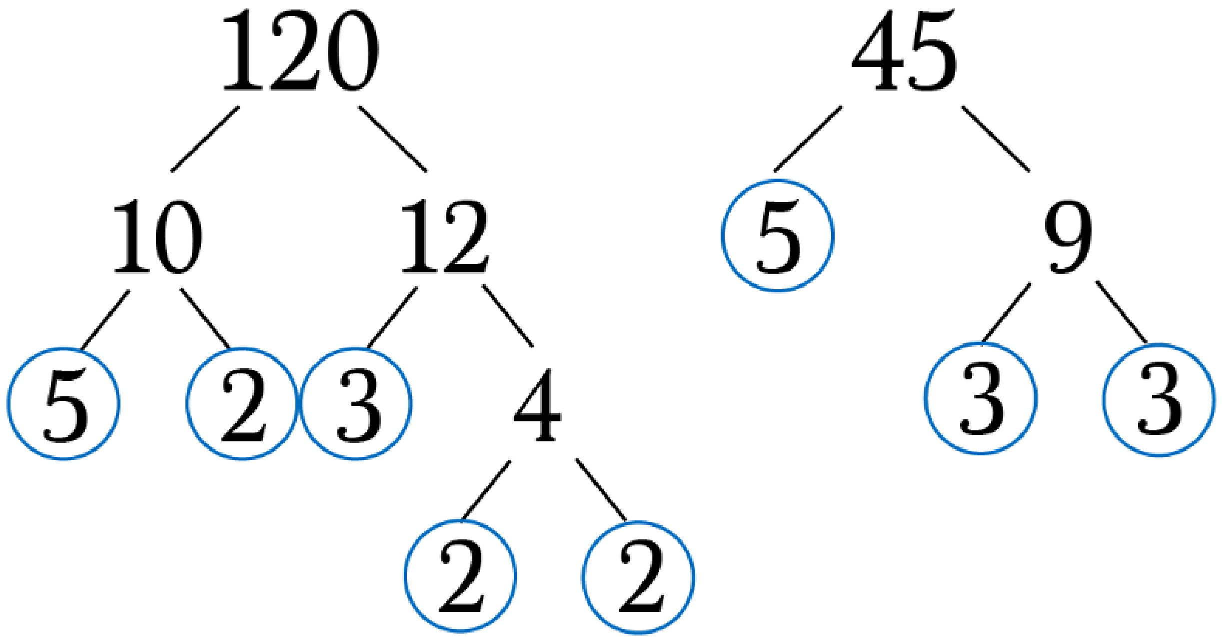


How many factors?

- 720
- 3640

Finding the Greatest CommonFactor

- The **GCF** is the largest number that divides into both values without a remainder. Let’s find the **GCF of 120 and 45.**



- **Step 2:** Write out the prime factorizations for each.

$$2^3 \cdot 3 \cdot 5 \text{ and } 3^2 \cdot 5$$

•**Step 3**: The **GCF will be the prime factors that are common to both factorizations multiplied together**. In this example, both factorizations have one 3 and one 5, therefore the GCF is 3 x 5 or 15.

$$\text{gcf}(120, 45) = 3 \cdot 5 = 15$$

- •*Note: The Greatest Common Factor and Greatest Common Divisor are exchangeable terms.*

GCF for 48 and 90

The LCM, least common multiple, is smallest value that two or more numbers multiply into. Let's find the LCM of 120 and 45.

- Begin by using factor trees to write out each number's prime factorization. We have already found the prime factorizations for 120 and 45:

$$2^3 \cdot 3 \cdot 5 \quad \text{and} \quad 3^2 \cdot 5$$

- The **LCM will be the product of the largest multiple of each prime that appears on at least one list.** For example we have a 2, 3 and 5, so I'll choose the largest multiples of each and find their product.

$$2^3 \cdot 3^2 \cdot 5 = 8 \cdot 9 \cdot 5 = 360$$

- Therefore the least common multiple of 120 and 45 is 360.

Use the prime factorizations to find GCF and LCM of 28, 49, and 63.

- At a summer camp, chocolate milk is served every other day, corn is served every 4 days and pizza every 7 days. Today all three were served. What is the smallest number of days until all three are served again?

- The organizers of a gymnastics event wish to arrange the participants in neat rows. They try rows of 2, 3, 4, 5, 6, 7 and 8, but in each case there is one gymnast left over. There are fewer than 1000 gymnasts in all. How many are there? Explain your reasoning. (Hint: What if 1 gymnast left the room?)

Basic Counting Rule

- If we are asked to choose one item from 2 separate categories where there are m items in the first category, n item in the second category.

The total number of available choices is m x n .

Example: There are three types of cones at the ice-cream shop and 10 different flavors. If a child can chose one type of cone and one flavor, how many choices are available to this child.

$$3 \text{ X } 10 = 30$$

\# of types of cones X # ice-cream flavors

The fundamental counting principle

- If there is a sequence of Independent events that can occur:
  - $a_1, a_2, a_3, \ldots\ldots a_n$ *ways* ,
  - Then the number that all events occur is
  - $a_1 \times a_2 \times a_3 \times \ldots\ldots a_n$

  - How many ways students can answer 3 questions true, false or I don't know.

  - The counting principle tells us, that since we can answer three ways every time
  - The number of ways students can answer is = 3 x 3 x 3 = 27

How many passwords are possible by using 6 digits where the first 2 digits must be letters and the last four digits must be numbers?

- 26 X 26 X 10 X 10 X10 X10 = 6,760,000

A restaurant offers a special menu where people can choose one of each different category.
People can choose: one of 4 beverages, one of 5 appetizers, one of 6 main dishes and one of 5 desserts.
How many different meals are possible?

A gate has a key pad with digits 0 to 9. How many possible code combinations are there if the code is 4 digits long?

- A) If repetition of numbers is allow?

10 x 10 x 10 x 10 = 10,000

- B) If repetition is not allow?

10 X 9 X 8 X 7 = 5040

Let’s talk about factorials

- $n! = 1 \times 2 \times 3 \times \ldots\ldots \times n$
- $5! = 1 \times 2 \times 3 \times 4 \times 5$
- $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

- Problem 11

Let’s talk about permutations

Permutation is a mathematical calculation of the number of ways a particular set can be arranged, where order of the arrangement matters.

$_n\mathbf{P}_r$ **=** n X (n-1) x (n-2) X ……. (n – r + 1)

$_n\mathbf{P}_r = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$

How many ways 10 athletes can be awarded 1st , 2nd and 3rd place?

$_{10}\mathbf{P}_3 = \frac{\mathbf{10}!}{(\mathbf{10}-\mathbf{3})!} = \frac{\mathbf{10}\ \boldsymbol{x}\ \mathbf{9}\ \boldsymbol{x}\ \mathbf{8}\ \boldsymbol{x}\ \mathbf{7}\ \boldsymbol{x}\ \mathbf{6}\ \boldsymbol{x}\ \mathbf{5}\ \boldsymbol{x}\ \mathbf{4}\ \boldsymbol{x}\ \mathbf{3}\ \boldsymbol{x}\ \mathbf{2}\ \boldsymbol{x}\ \mathbf{1}}{\mathbf{7}\ \boldsymbol{x}\ \mathbf{6}\ \boldsymbol{x}\ \mathbf{5}\ \boldsymbol{x}\ \mathbf{4}\ \boldsymbol{x}\ \mathbf{3}\ \boldsymbol{x}\ \mathbf{2}\ \boldsymbol{x}\ \mathbf{1}}$ **=** ***10 x 9 x 8***

Combinations:

- We say that there are $_n\mathbf{C}_r$ **combinations** of size **r** that may be selected from among n choices without replacement where order doesn't matter.

$$_n\mathbf{C}_r = \frac{n\mathbf{P}_r}{r\mathbf{P}_r} = \frac{n!}{(n-r)!\,r!}$$

In a lottery a player picks 4 number from 0 to 9 (without repetition) . How many different choices does the player Have?

a) Order matter:

$$_{10}\mathbf{P}_4 = \frac{10!}{(10-4)!} = 5040$$

b) Order does not matter

$$_{10}\mathbf{C}_4 = \frac{10!}{(10-4)!\,4!} = 210$$

Let’s assume that we have 10 balls, and let us say that balls 1, 2, 3 are chosen.
These are the possibilities

- Order does matter:

- 123
- 132
- 213
- 231
- 312
- 321

The permutations have 6 times as many possibilities.

- Order does not matter:

- 123

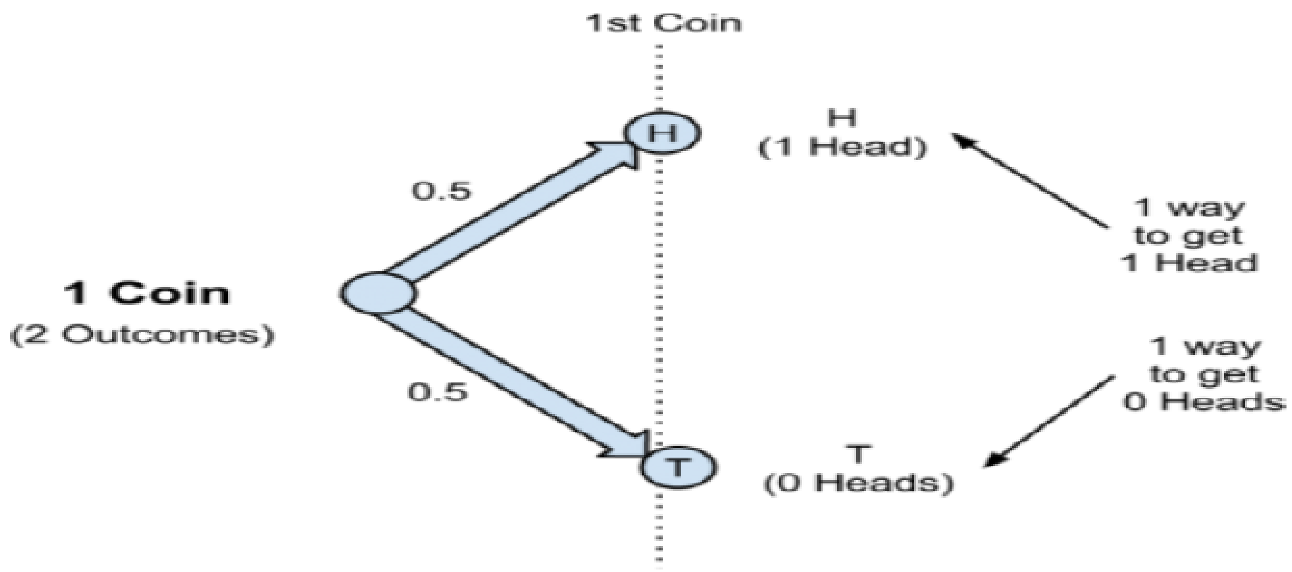
Combination problems:

- How many ways 5 students can be chosen from 12 students?

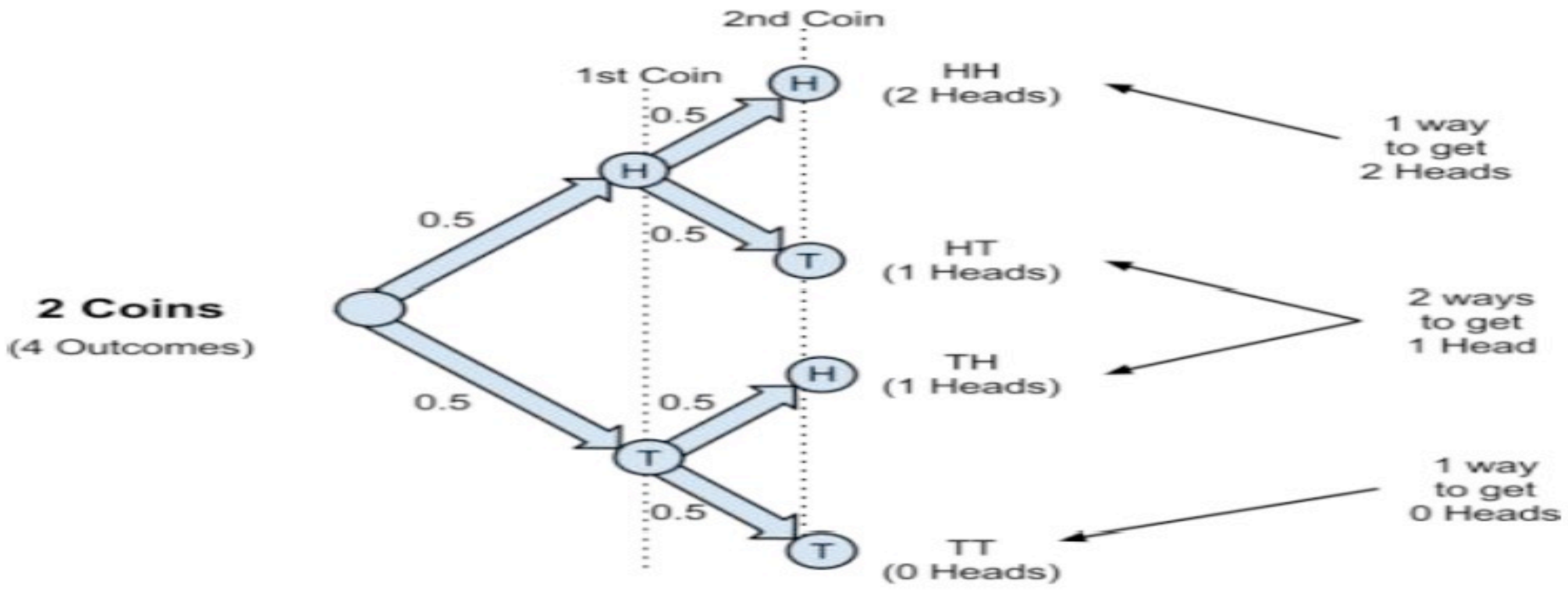
- There are 10 people at a party who shakes hands with each other. If each two of them shake hands with each other, how many handshakes happen at the party?

What is probability

- Probabilities are decimal numbers or fractions between 0 and 1. The higher the probability (meaning the closer to 1), the more likely it is that whatever we're talking about will actually happen.
- When we toss a single coin there are exactly 2 possible outcomes—heads or tails—which we'll abbreviate as "H" or "T."

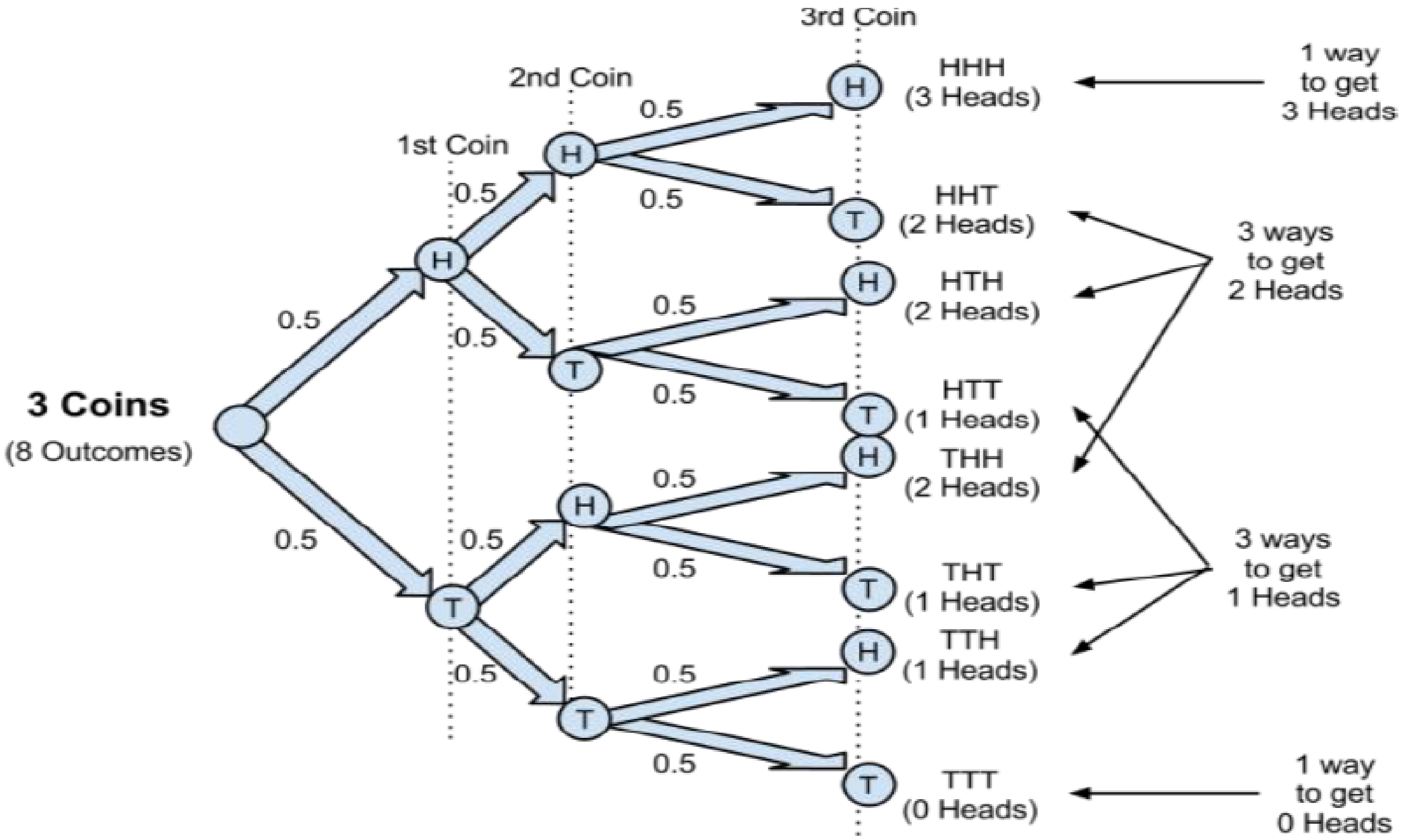


- Thinking about 1 coin is almost too easy, so let's move on to 2 coins

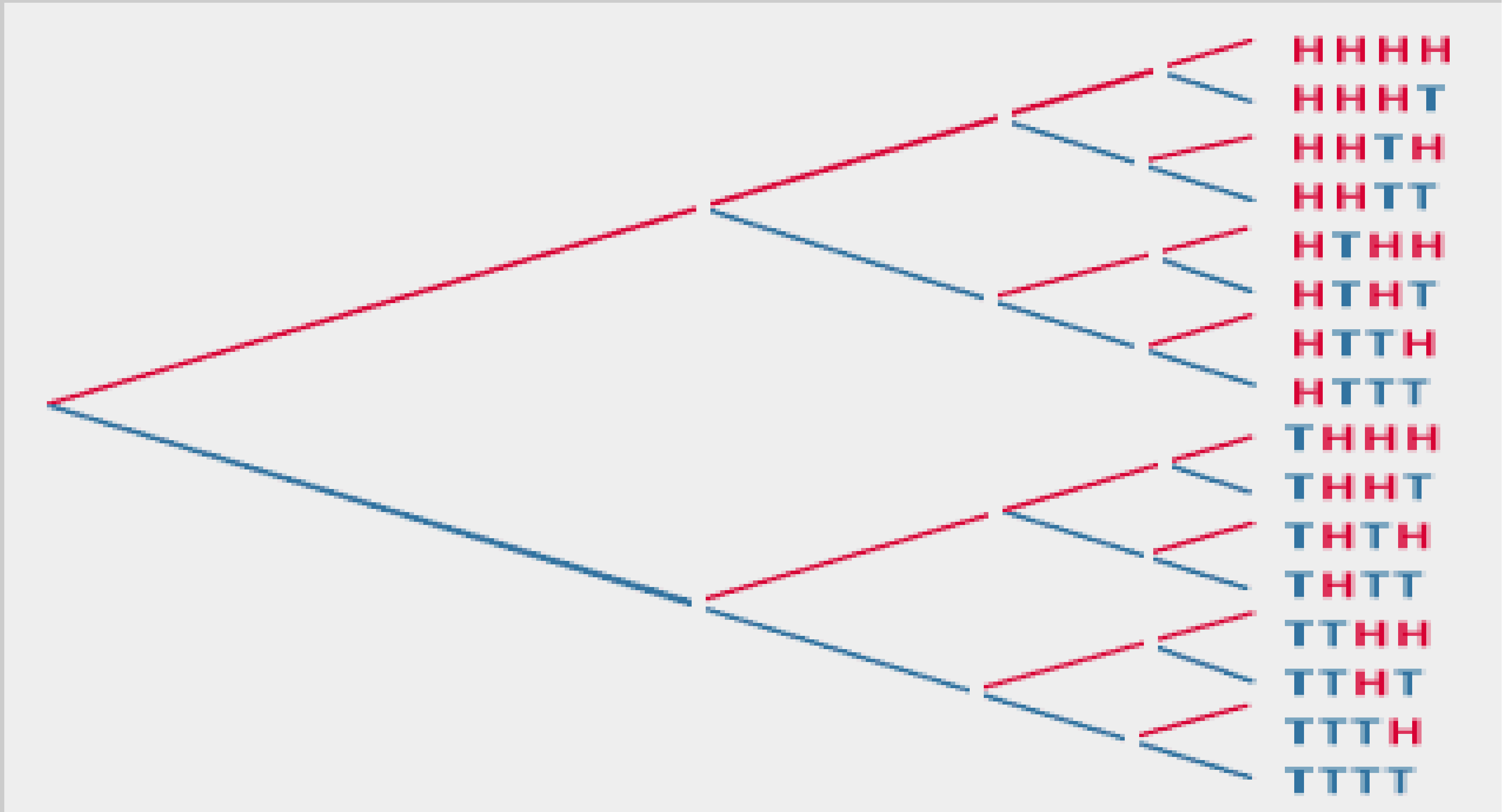


- There are 4 possible outcomes when tossing 2 coins. And there are! HH, HT, TH, or TT.

• Let's try tossing 3 coins at once.



4 coin toss


HHHH
HHHT
HHTH
HHTT
HTHH
HTHT
HTTH
HTTT
THHH
THHT
THTH
THTT
TTHH
TTHT
TTTH
TTTT

A poker hand consists of five cards randomly dealt from a deck of 52 cards. The order of the cards does not matter. Determine the following probabilities:

- Determine the probability that exactly 4 of these cards are Aces:

P(4 aces and 1 no ace)

- **${}_{52}C_5$ = is the number of ways 5 cards can be dealt from a deck with 52 cards**
- **${}_{4}C_4$ = There are only 4 aces, so this is the number of ways we choose 4 aces out of 4.**
- **${}_{48}C_1$ = There are 48 cards that are no aces. This will give us the number of ways we can to choose one card , no ace.**

- P(4 aces and 1 no ace) = $\frac{{}_{4}C_{4} \times {}_{48}C_{1}}{{}_{52}C_{5}} = \frac{1 \text{ x } 48}{2598960}$

Probability that all cards are hearts

- P(5 hearts)
- **$_{52}C_5$ = is the number of ways 5 cards can be dealt from a deck with 52 cards**
- **$_{13}C_5$ = There are only 13 hearts. How many ways we choose 5 cards out of 13 heart cards.**

- P(5 hearts) = $\frac{_{13}C_5}{_{52}C_5}$

Determine the probability of selecting 2 Queens and 2 Kings

- P(2 Queens and 2 kings)
- **${}_{52}C_5$ = is the numbers of ways that 5 cards can be dealt from a deck with 52 cards**
- **${}_{4}C_2$ = There are only 4 Queens, this is the number of ways we choose 2 out of 4.**
- **${}_{4}C_2$ = There are only 4 Kings, this is the number of ways we choose 2 out of 4.**
- **${}_{44}C_1$ = There are 44 cards that are not Queens and Kings, this is the number of ways we choose 1 card out of 44.**

- P(2 Queens and 2 kings) = = $\dfrac{{}_{4}C_2 \times {}_{4}C_2 \times {}_{44}C_1}{{}_{52}C_5}$

Sources:

- https://www.youtube.com/watch?v=qJ7AYDmHVRE
- https://www.youtube.com/watch?app=desktop&v=ZKrz2t8EYlU