ELMACON 2015 Preparation March 28th

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Fundamental Counting Principle:

When a result consists of separate parts, we can multiply these parts together to find the total number of ways the result can be obtained.

Example 1: A guy has 6 different colored shirts, 4 different pairs of pants and 2 different pairs of shoes. How many ways can he choose his outfit if he needs one shirt, one pant and one pair of shoes?

Factorials: ex: 5! = 5x4x3x2x1 = 120

The number of distinct arrangements of 3 objects is 3x2x1 = 3!=6
The number of distinct arrangements of 5 objects is 5!=120

Arrangement of n Objects:

The number of distinct arrangements of n objects can be expressed as n! = n(n-1)(n-2)...(3)(2)(1)

Permutations are the number of ways that n distinct things can be arranged if they things are taken r at a time. We write: nPr = n(n-1)(n-2)...(n-r+1) = n!/(n-r)!
This is just a generalization of the fundamental counting principle when repetitions are not allowed. Note that permutations apply only when repetitions are not allowed and when order is important.

Combinations are the number of ways that n distinct things can be arranged if the things are taken r at a time and the order of things is not important.
We write: nCr = (nPr)/r! = n!/(r!(n-r)!)
Combinations give the number of possible subsets. when repetitions are not allowed and order is NOT important.

Probability

Probability is a measure of the chance that an event will occur.
P(event) = (number of desired outcomes)/(total number of possible outcomes)

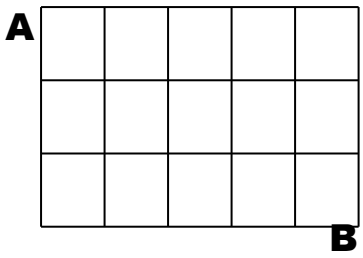
Example 2: How many ways can we select 3 winners for 1st, 2nd, and 3rd places for a race involving 10 runners?

Example 3: How many ways can we select 3 volunteers from a group of 10 students?

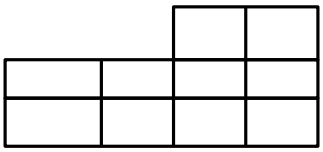
Example 4: A) How many ways can three couples line up for a photo?
B) What if each couple must stay together?
C) What is the probability for that to happen?

Example 5: Mr. Smith keeps the phone numbers for his 12 closest friends (4 male and 8 female) in his digital phone memory. A) How many ways can he list them? B) How many ways can he list them if no two men are listed next to each other? C) What is the probability for that to happen?

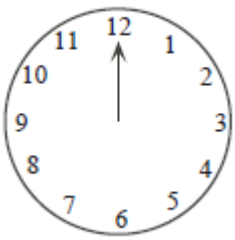
Example 6: a) How many Paths are possible from point A to point B if all motion must be to the right or downwards?
b) How many squares of all sizes are there?
c) How many rectangles?



Example 7: How many different rectangles are there altogether in the diagram?



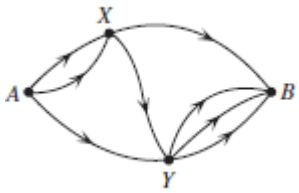
1 The clock shows the time of 12:00 (the hour hand and the minute hand point in the same direction). How much time (in hours) passes until both hands point again in the same direction? Express your answer as a common fraction.



2. In a club, the ratio of boys to girls was $\frac{13}{19}$. Then, 4 more boys joined the club and now the new ratio is $\frac{5}{7}$. How many boys are now in the club?

3. The measures of the sides of triangle A are 5cm, 5cm and 6cm. The measures of the sides of triangle B are 5cm, 5cm and 8cm. What is the ratio of the area of A to the area of B?

4. In how many ways can you walk from Point A to point B if you must walk along the directions marked by arrows?



5. Suppose that when a man is at point A, the probability that he walks along any of the three paths is $\frac{1}{3}$. If he is at point X the probability that he walks along any of the 2 paths is $\frac{1}{2}$. If he is at point Y, the probability that he walks along any of the three paths is $\frac{1}{3}$. Two men walk independently from point A to point B. What is the probability that both choose the same path?

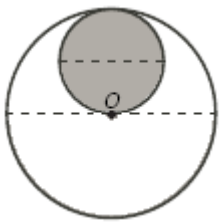
6. Five boys and two girls sit at random at a round table with 7 seats. What is the probability that the girls do not sit next to each other? Express your answer as a common fraction.

7. Every student in a class of 20 sent an e-mail to each of the other students of the class. How many e-mails were sent in total?

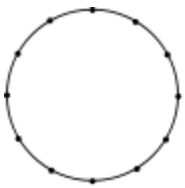
8. What is the value of 1002×998**?**

9. $3^{11} \times 3^{2014} = 3^{N \times 27}$**. What is the value of** N**?**

10. The two circles below are tangent. The point O is the centre of the big circle and is also on the circumference of the small circle. The measure of the circumference of the big circle is $32\sqrt{\pi}$**. What is the area of the shaded region?**

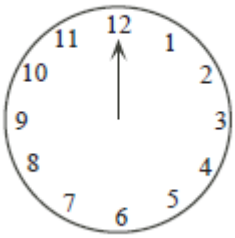


11. 12 points are equally spaced on a circle. How many non-congruent decagons (ten sided polygons) can be formed using any 10 of these 12 points as vertices?



12. An isosceles trapezoid is inscribed in a circle. Given that its bases are equal to 10 and 18, its height is equal to 4, and the center of the circle lies outside the trapezoid, find the radius of the circle.

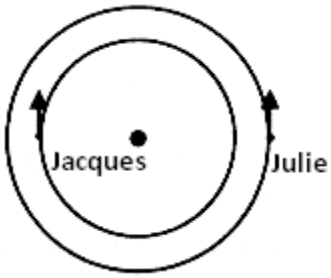
13. What is the acute angle (in degrees) between the hour hand and the minute hand at 3:30?



14. Three angles of a pentagon have measures 88^{o}, 124^{o}, and 92^{o}. If the measures of the remaining 2 angles are equal, what is the measure, in degrees, of one of the remaining angles?

15. The points $(0,0)$, $(2,7)$, and $(k,20)$ lie on a line. What is the value of k? Express your answer as a common fraction.

16. Jacques runs on the inner track. Julie runs on the outer track. The radius of the outer track is R. The radius of the inner track is r. For every 17 full circles that Julie completes, Jacques completes 19 full circles. Jacques' speed is $\frac{29}{31}$ of Julie's speed. What is the value of $\frac{R}{r}$? Express your answer as a common fraction.



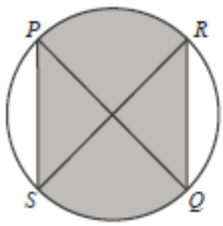
17. Together, taps A and B will fill the swimming pool in 6 hours. Tap C will fill the swimming pool alone in 15 hours and together with tap B in 5 hours. Alone, in how many hours will tap A fill the swimming pool?

18. How many seconds after noon will the minute and hour hand form a straight angle?

19. How many different three-digit numbers can be made using any three of the following five digits: 3, 4, 4, 5, and 5?

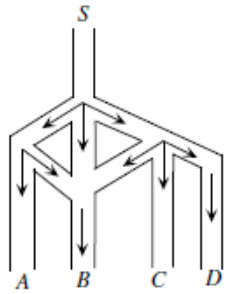
20. Find the radius r of a circle inscribed in a triangle with side lengths 6, 8, and 10 respectively.

21. $PQ \perp RS$ are both diameters of length 10 cm. What is the area of the shaded region rounded to the nearest whole number of cm^2?



22. In a race over d m, A would beat B by 20 m; B would beat C by 10 m, and A would beat C by 28 m. What is the value of d?

23. The figure below describes the flow of water through a system of pipes (all the water flows as indicated by the arrows). In any 3-way split $\frac{1}{3}$ of the water continues in any of the 3 branches, and in any 2-way split $\frac{1}{2}$ of the water continues in any of the 2 branches. What fraction of the flow from S does not end up at B?



24. If you fill the missing numbers in the magic square below, all horizontal, vertical and diagonal sums are the same. What is the value of x?

$2x$	3	2
		-3
0	x	

25. Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. What is the ratio of the area of the other small right triangle to the area of the square?