ELMACON Preparation Session

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April 8, 2014

1 Problem Set B

1. Use the following information to find the number N: On a certain island, it never rains all day. If it rains in the morning, it is clear in the afternoon. If it rains in the afternoon, it was clear in the morning. On some days it doesn't rain at all. During a recent period of N days there were 11 clear afternoons, 11 clear mornings, and 10 days when some rain fell.

1. ⎯⎯⎯⎯⎯⎯⎯⎯

2. In the division problem $(11^{10} - 1) \div 100$, what is the remainder? (Note that 11^{10} means $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$).

2. ⎯⎯⎯⎯⎯⎯⎯⎯

3. How many members are there in the StayFit club? Clues: There are 40 members who swim, 30 members who play tennis, and 20 members who jog. Five of the members swim and play tennis but do not jog, five swim and jog but do not play tennis, and 5 play tennis and jog but do not swim. Four members do all three activities; all members do at least one.

3. ⎯⎯⎯⎯⎯⎯⎯⎯

4. There are 9 teams in a soccer league. Each team plays each other team four times. How many games are played?

4. ⎯⎯⎯⎯⎯⎯⎯⎯

5. Use the following information to find the number N: N is an even three-digit number. N equals the area of a square whose sides are whole numbers; when the length of one side is written down, its digits add up to 4.

5. ⎯⎯⎯⎯⎯⎯⎯⎯

6. Consider the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many different subsets containing three elements each have elements that sum to 10?

   Background: Both $\{0, 4\}$ and $\{1, 3\}$ are subsets containing two elements that add up to 4: these are the only two-element subsets within this sum. Repeated elements are not allowed in a subset, so $\{2, 2\}$ may not be used. Also, the order of writing makes no difference so $\{1, 3\}$ and $\{3, 1\}$ are considered the same subset.
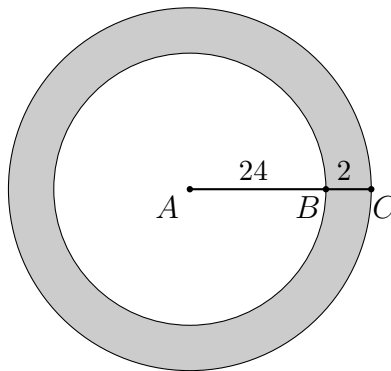
   6. _____

7. A restaurant offers a choice of 3 meat items, 3 vegetable items, and 3 desert items. How many different meals can Jane order, if she chooses 2 different meat items and 2 different non-meat items, of which at least one is a vegetable?

   7. _____

8. The dimensions in the sketch below are in meters. Find the area of the shaded part, to the nearest whole number of square meters.

   Notes: Both figures are circles centered at A. The distance from A to B is 24 meters. The distance from B to C is 2 meters. (The diagram is not to scale).

   8. _____